INGENIERÍA CIVIL

Programa de Doctorado en Ingeniería

Línea de Investigación

Geotecnia y Riesgos Geoambientales

50 AÑOS POSGRADOS INGENIERÍA CIVIL

Área Curricular de Ingeniería Civil y Agrícola
Facultad de Ingeniería
Sede Bogotá

UNIVERSIDAD NACIONAL DE COLOMBIA

Reliability Assessment of Wedge Failure in Rock Slopes by Random Sets Theory

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- Wedge Failure
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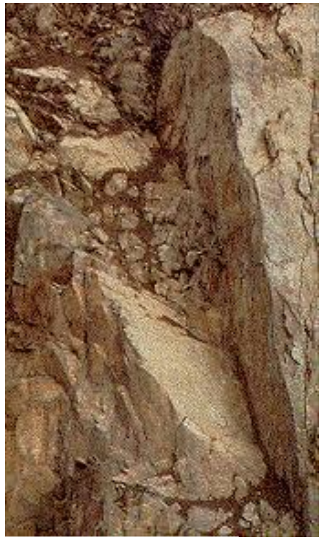
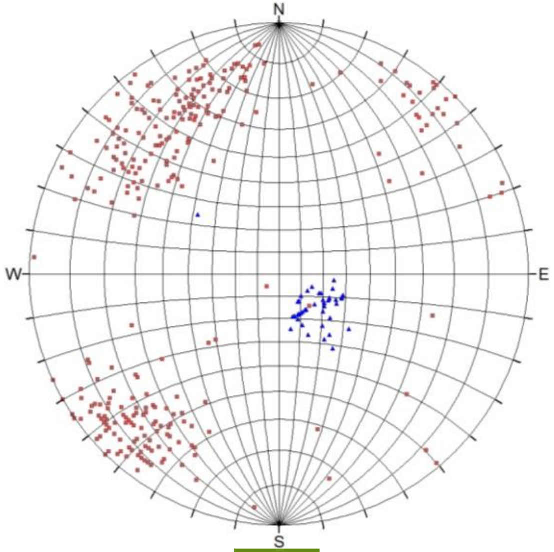
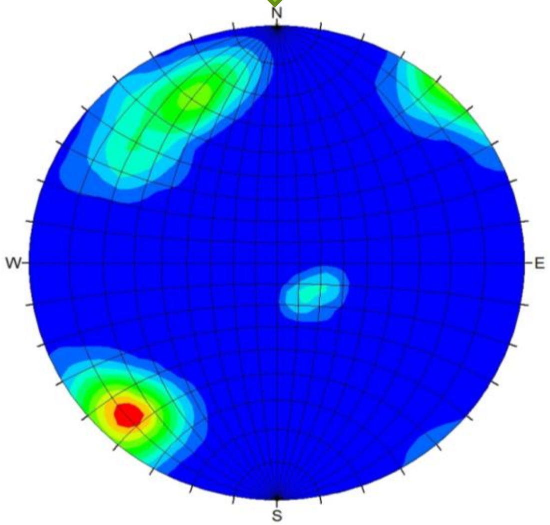
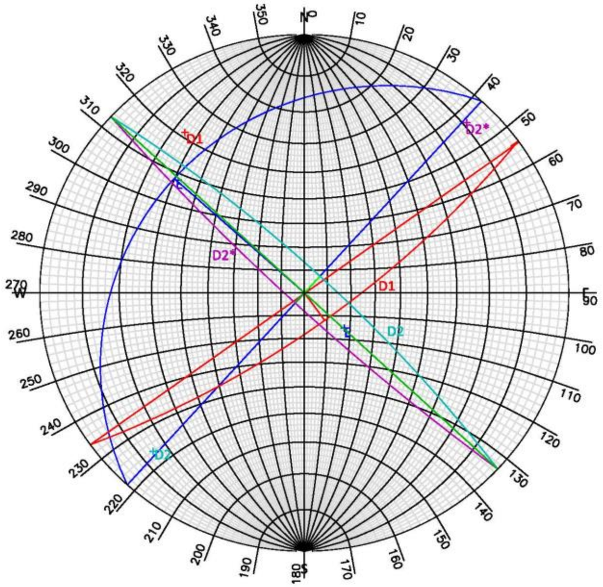
Motivation

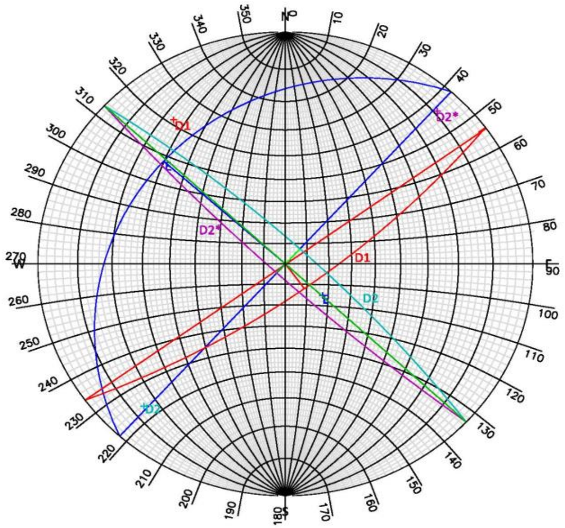




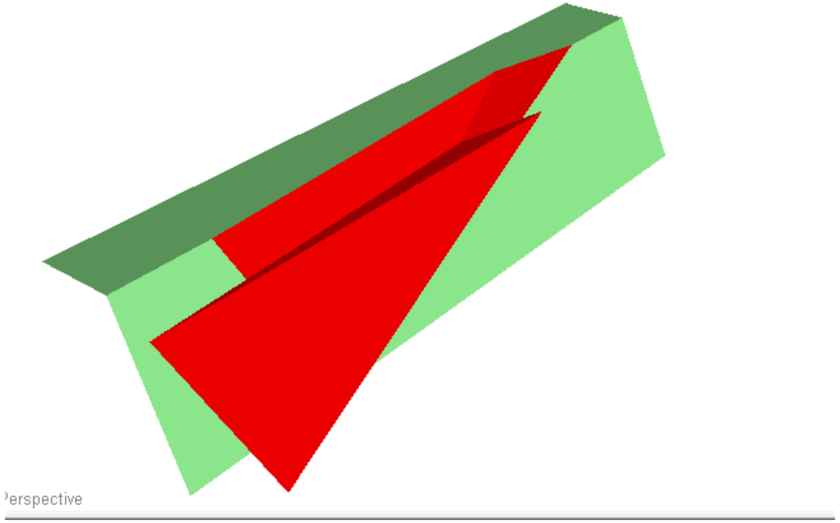




Motivation



C, Φ*



FoS

Motivation

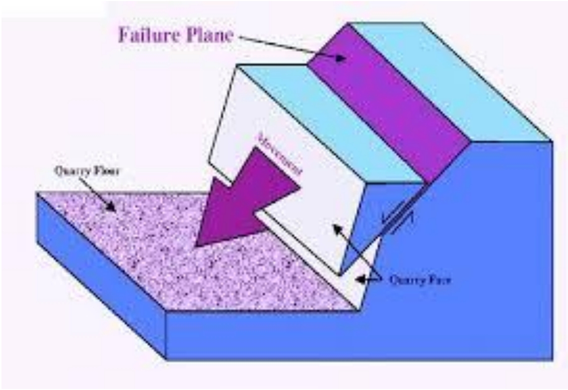
- Variability of properties, both geometrical and mechanical (Uncertainty)
- Several joint geometry information at different locations
- Limited information on mechanical parameters (financial and technical constraints)
- Uncertainty is considered by reliability assessment
- Probability distribution is required

Hence

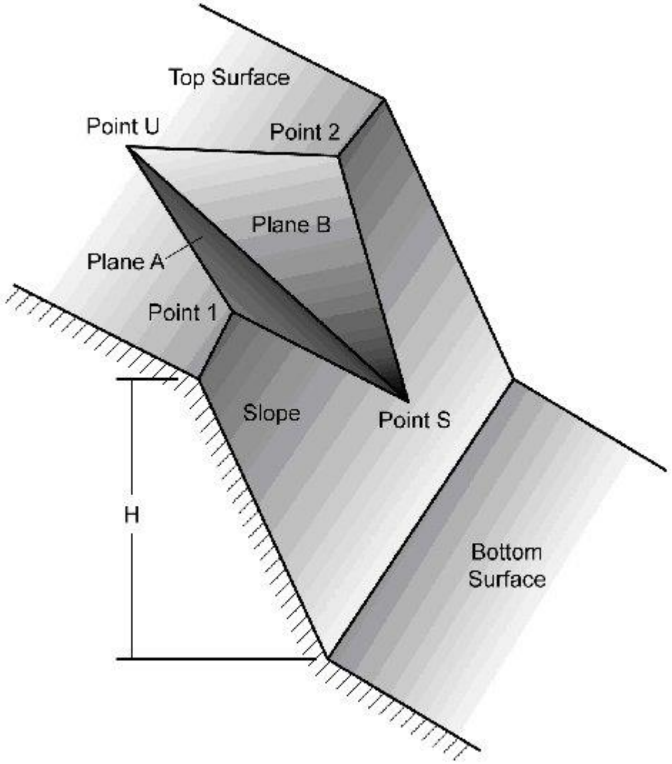
- Alternative formulation to perform reliability assessment under limited information
- Random sets theory provides this alternative approach to deal with uncertainty when limited info is available

Discontinuities

- **Presence of discontinuities is associated with:**
  - **Scale**
  - **Stress and displacements jumps across the fractures**
  - **Relative motion of blocks delimited by discontinuities**
  - **In many case, discontinuities control the geometry of failure**

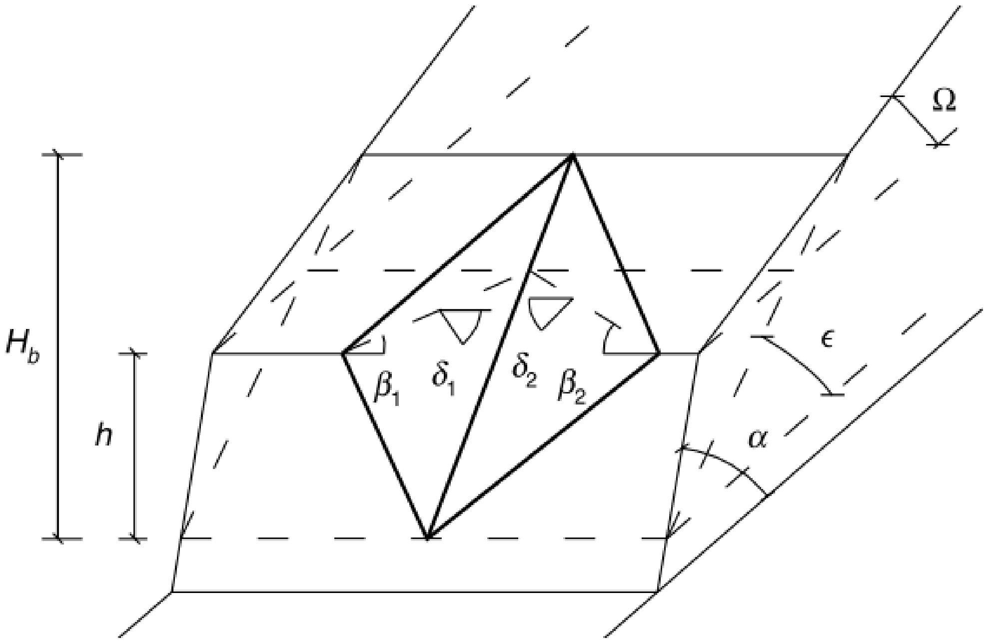


Wedge failure



Wedge Failure Model

- **Low's model**

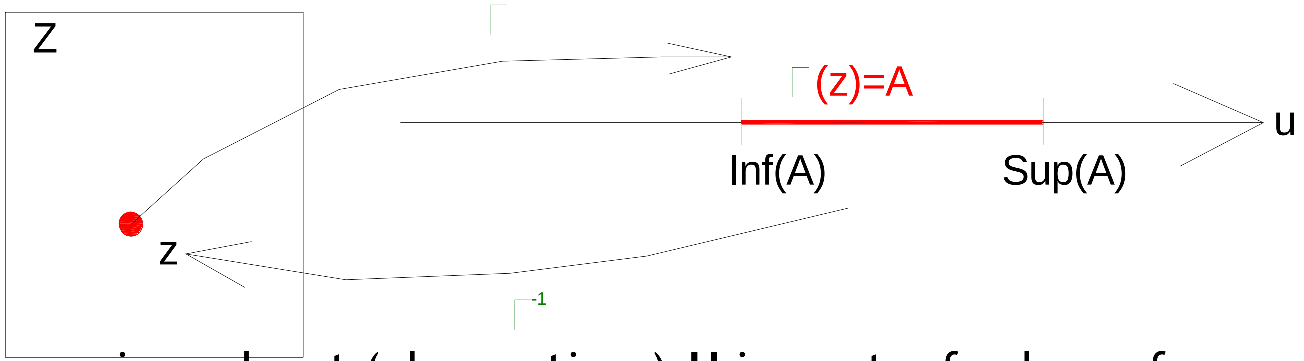


$$FS = \left(a_1 - \frac{b_1 G_w}{S_\gamma}\right) * tan\emptyset_1 + \left(a_2 - \frac{b_2 G_w}{S_\gamma}\right) * tan\emptyset_2 + 3b_1\frac{c_1}{\gamma h} + 3b_2\frac{c_2}{\gamma h}$$

Imprecise Probabilities

- Based on interval analysis
- RST involves both intervals concepts and probabilities
- Random sets theory provides this alternative approach to deal with uncertainty when limited info is available

Random sets theory



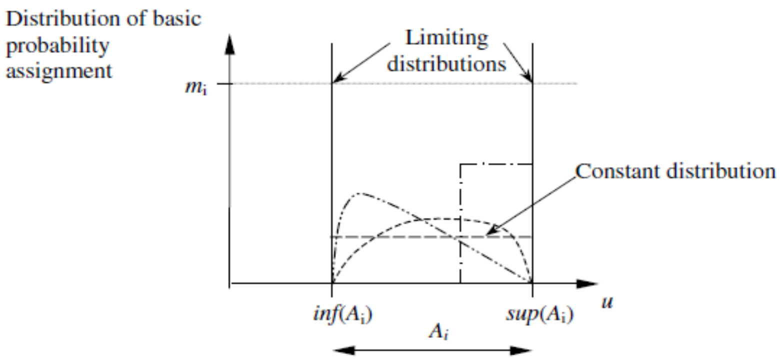
Z is a universal set (observations) **U** is a set of values of measurements
Γ is a multivalued mapping

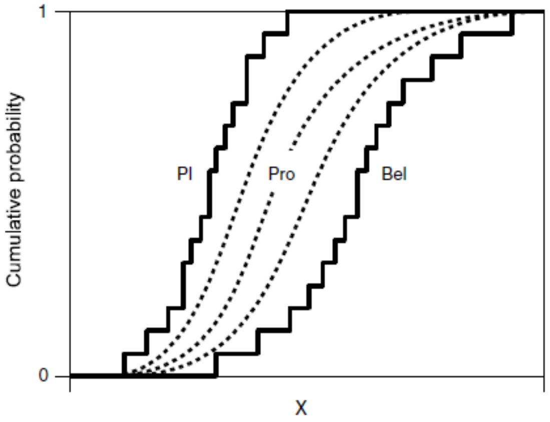
$$\boldsymbol{\varphi} = \{A_i : i = 1, \ldots, n\} \quad \text{Support of ramdon set}$$

Random set on Z is a pair (**φ, m**), where **m** (known as the basic probability assignment) is a frequency function which can be defined as:

$m(A_i) \rightarrow [0,1]$ $\quad$ $\mathrm{m}(\emptyset) = 0$ $\quad$ $\sum_{A \in \varphi(U)} m(A) = 1$

Random sets theory





Bel(E)≤Pro(E)≤Pl(E)

$$Bel(E) = \sum_{A_i : A_i \subset E} m(A_i)$$

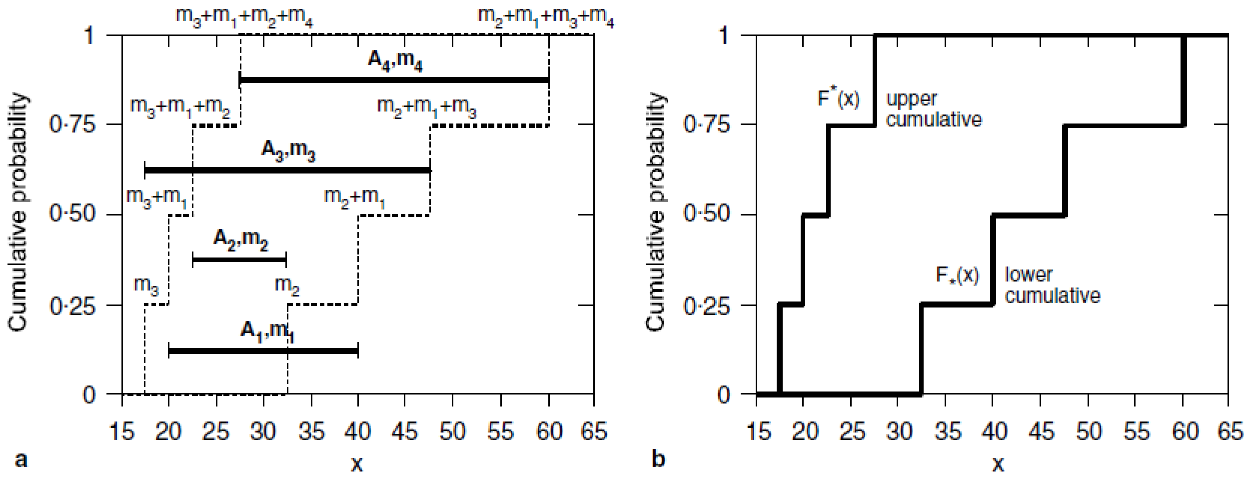
$$Pl(E) = \sum_{A_i : A_i \cap E \neq \emptyset} m(A_i)$$

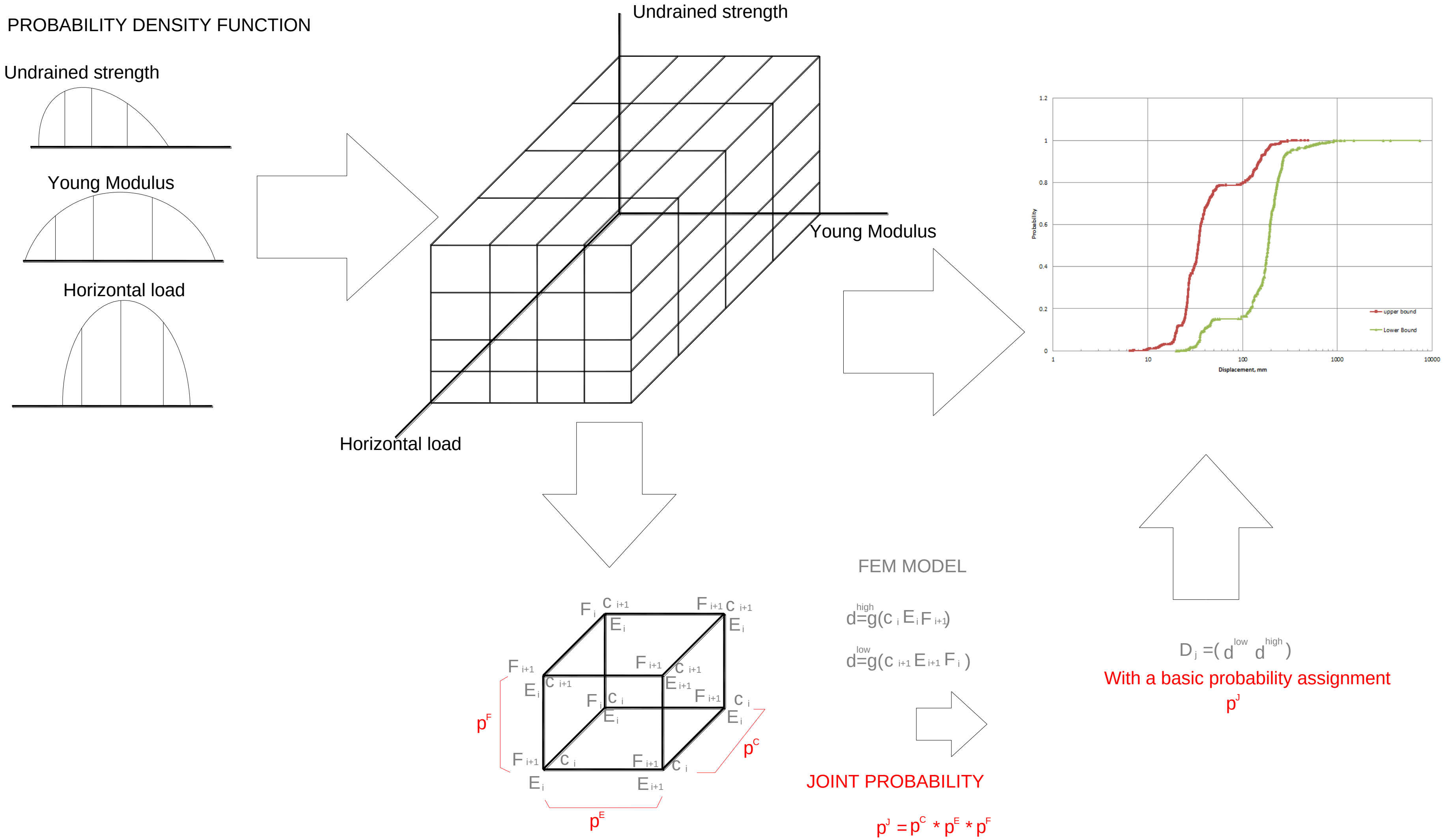
Random sets theory

If the focal set A_i is a closed interval of real numbers: $f(A_i)=\{ x / x \in [l_i, u_i] \}$, then the lower $F_*(x)$, and upper, $F^*(x)$, probability mass function, at some point **x** can be obtained as follows:

$$F_* = \sum_{i: x \geq u_i} m(A_i)$$

$$F^* = \sum_{i: x \geq l_i} m(A_i)$$




PROBABILITY DENSITY FUNCTION
Undrained strength
Young Modulus
Horizontal load
Undrained strength
Young Modulus
Horizontal load
Probability
Displacement, mm
upper bound
Lower Bound
FEM MODEL
d^high=g(c i E i F i+1)
d^low=g(c i+1 E i+1 F i)
JOINT PROBABILITY
p^J = p^C * p^E * p^F
D i =(d^low d^high)
With a basic probability assignment
p^J
p^F
p^C
p^E

Probability function computation (upper)

1. Assume that probability mass of each interval is concentrated at the lower bound of each focal element
2. Sort lower bound from the smallest to the greatest
3. Define interval from sorted data
4. Assign probability concentrated on the low bound of the original focal element to each new interval
5. Sum up the probability assigned to each interval

Model inputs

- Wedge model random input parameters expressed as random set

Joint 1													
Dip direction			Probability Assignment 1	Probability Assignment 2	Dip			Probability Assignment 1	Probability Assignment 2	ID	Friction angle 1		Probability Assignment
ID	Lower	Upper			ID	Lower	Upper						
1	91	93	0,33	0,05	1	46	47	0,33	0,05	**1**	Lower	Upper	
2	92	95	0,33	0,05	2	49	51	0,33	0,05	**2**	30	32	0,5
3	101	103	0,34	0,9	3	50	52	0,34	0,9	**3**	27	29	0,5

Joint 2													
Dip direction			Probability Assignment 1	Probability Assignment 2	Dip			Probability Assignment 1	Probability Assignment 2	ID	Friction angle 2		Probability Assignment
ID	Lower	Upper			ID	Lower	Upper						
1	189	191	0,33	0,05	1	27	29	0,33	0,05	**1**	Lower	Upper	
2	190	193	0,33	0,05	2	31	33	0,33	0,05	**2**	16	17	0,5
3	192	195	0,34	0,9	3	26	30	0,34	0,9	**3**	20	22	0,5

- Wedge model deterministic input parameters

Slope	
Dip direction	140
Dip	65
Height	30
Upper slope ar	12
Rock mass	
c1 (kPa)	24
c2 (Kpa)	48
$s\gamma$	2,5
γ (kN/m3)	25

- Wedge model probabilistic input parameters

Parameter	Joint 1		Joint 2	
	Mean	Standard deviation	Mean	Standard deviation
Dip direction	95,8	5,0	191,7	2,2
Dip	49,2	2,3	29,3	2,6
Friction angle	29,5	2,1	18,8	2,8

Reliability assessment

Limit state function g(x):

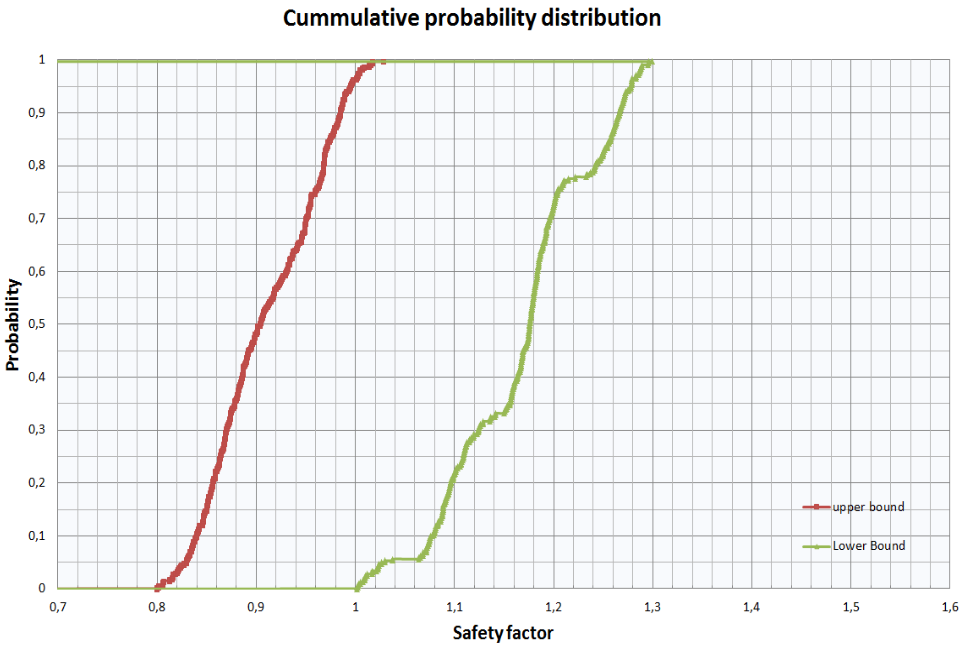
$$g(x) = \begin{cases} > 0 \; for\ the\ safe\ state\ of\ the\ structure \\ < 0 \; for\ the\ failure\ of\ the\ structure \end{cases}$$

$X = (X_1, X_2, \ldots, X_n)$ Random variables

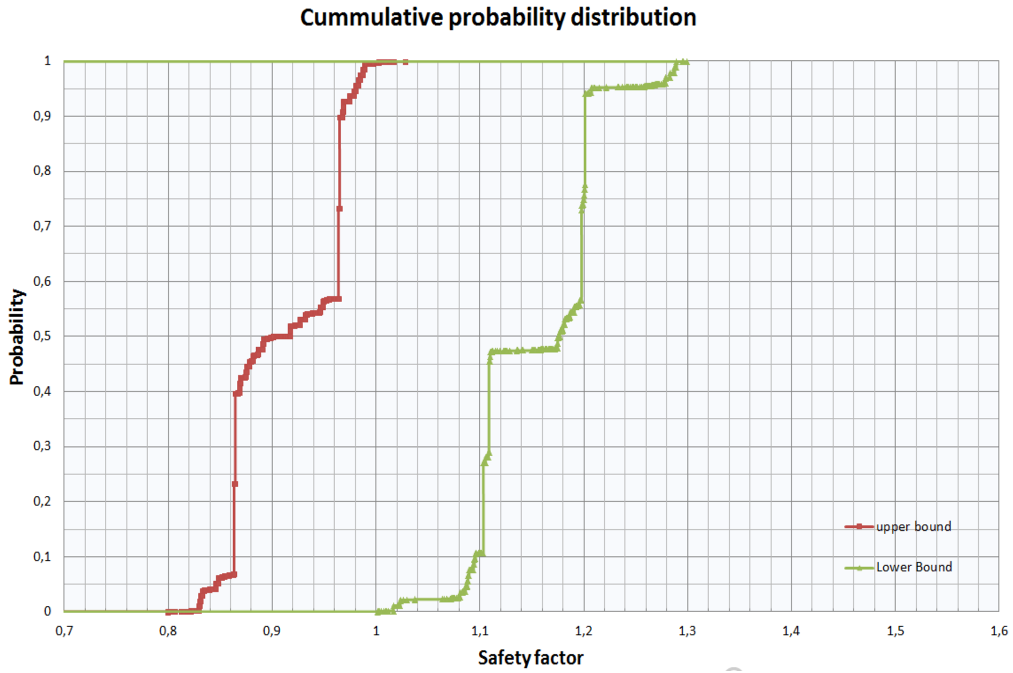
$$\mathrm{g(x)} = \alpha^* - \mathrm{\alpha}(\mathrm{X}_c, X_E, X_F)$$

$$\mathrm{g(x)} = \boldsymbol{FOS^C} - \boldsymbol{FOS^*}$$

Results

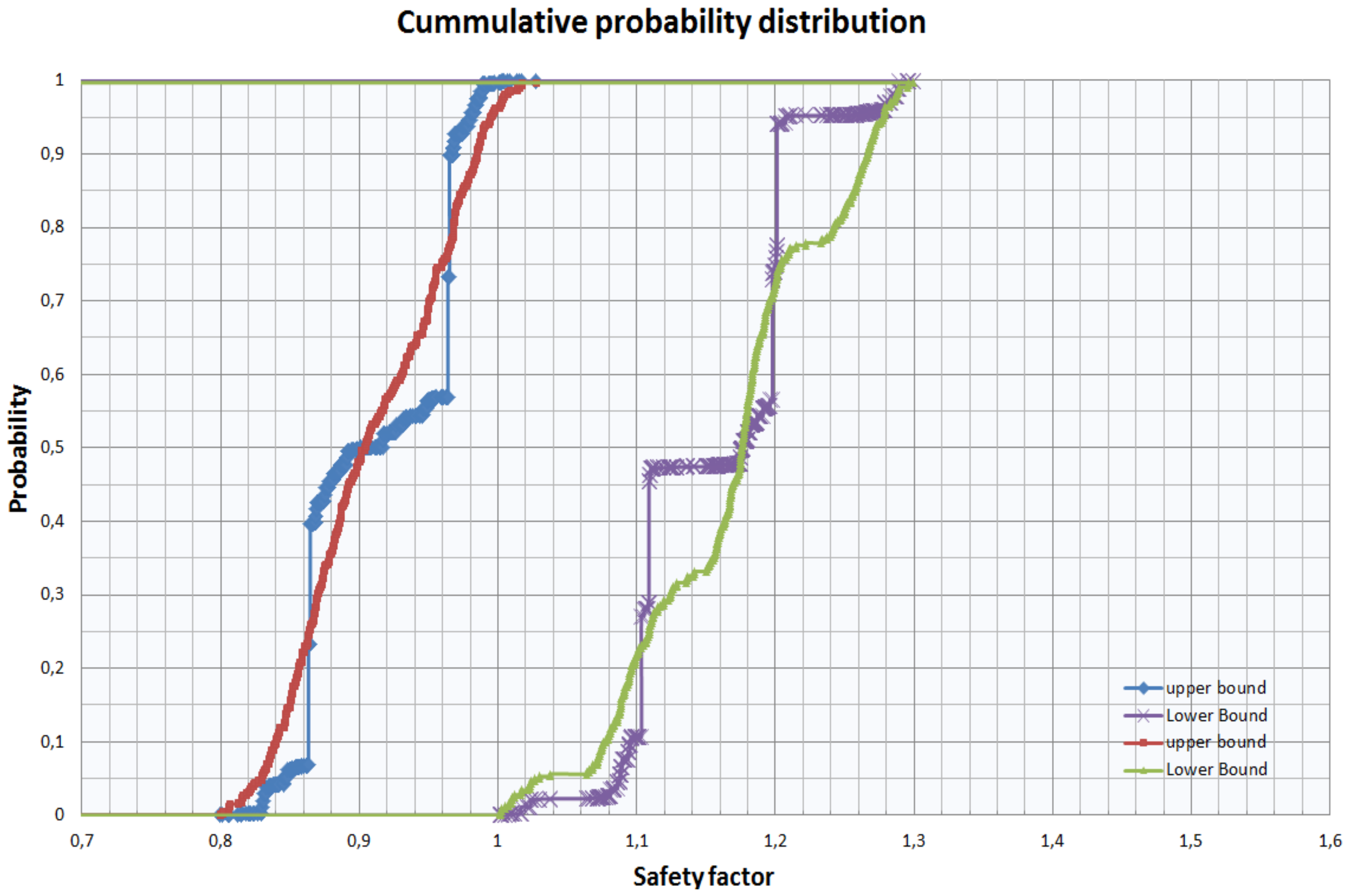


Bounded cumulative probability distribution of the factor of safety, calculated by random sets with the same probability assignment



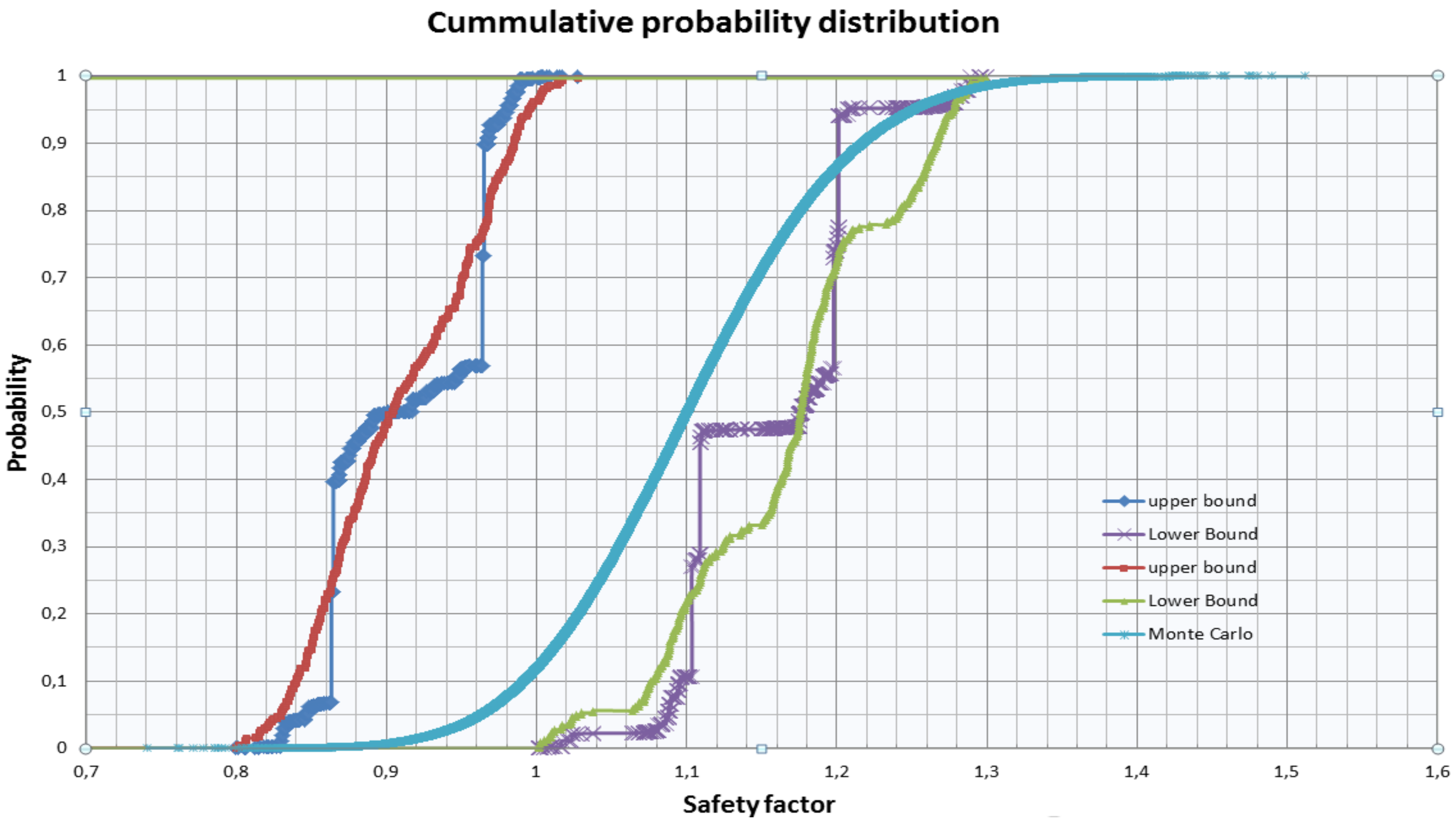
Bounded cumulative probability distribution of the factor of safety, calculated by random sets with assigning 90% of the probability to one interval

Results



Cummulative probability distribution
Probability
Safety factor
1
0,9
0,8
0,7
0,6
0,5
0,4
0,3
0,2
0,1
0
0,7
0,8
0,9
1
1,1
1,2
1,3
1,4
1,5
1,6
upper bound
Lower Bound
upper bound
Lower Bound

Results



Cummulative probability distribution
Probability
Safety factor
0
0,1
0,2
0,3
0,4
0,5
0,6
0,7
0,8
0,9
1
0,7
0,8
0,9
1
1,1
1,2
1,3
1,4
1,5
1,6
upper bound
Lower Bound
upper bound
Lower Bound
Monte Carlo

Conclusion

- Random sets generate a bounded cumulative probability distribution of the model, considering the uncertainty of the input data, even when limited information is available. The distribution of the factor of safety of the wedge demonstrate that RST is suitable to be implemented in rock mechanics problems, in which there are limited number of input data i.e. laboratory results and joints geometry information.
- RST shows probable combination of input data that lead to an unsatisfactory response of the system. This information can be helpful for practitioners when they face design decision, since they can see explicitly the response of the worst case scenario
- RST approach allows computing bounded probability distribution with a smaller number of calculations than Monte Carlo simulation. This feature makes the RST suitable to apply along with more complex and computationally more expensive modeling techniques like FEM or DEM.

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